

BRIEF COMMUNICATION

A METHOD FOR MEASURING TRANSPARENT DROPLET DIAMETERS†

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A pervasive task in many studies of two phase problems is that of measuring the diameter of transparent liquid droplets. This can be done with good accuracy if backlighting is possible. But circumstances often preclude backlighting. When, for example, droplet motions are to be studied, one must illuminate the drop stroboscopically from the side (or at an angle) with the background kept black. In such cases the droplet outline often cannot be seen. Only two bright spots appear on the equator, where the drop should be on a photograph.

Consider figure 1 which shows light impinging on the equatorial plane of a droplet at an angle θ off the normal to the camera axis. Two additional angles α and β are identified on the sketch. Our aim is to determine where the two spots are located relative to the unknown outline of the drop.

We see immediately that

$$\alpha = \beta + \theta/2 - \pi/4. \quad [1]$$

Furthermore Snell's law tells us that

$$n \sin \beta = \sin (\pi/2 + \alpha - \theta) \quad [2]$$

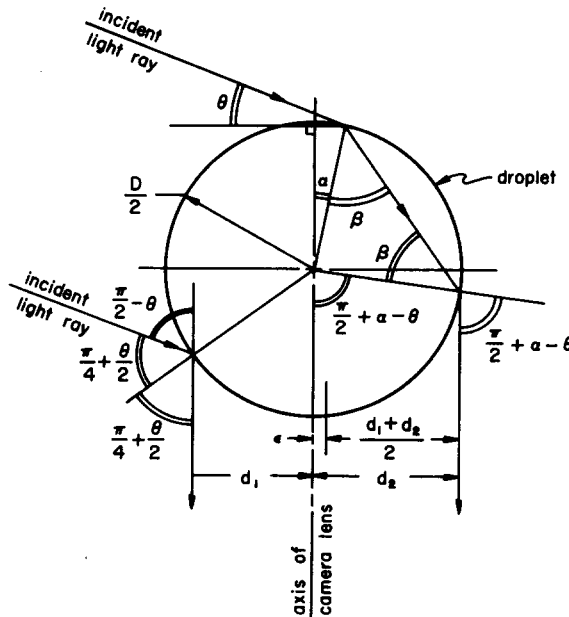


Figure 1. Diagram of droplet and light rays, and definition of terms.

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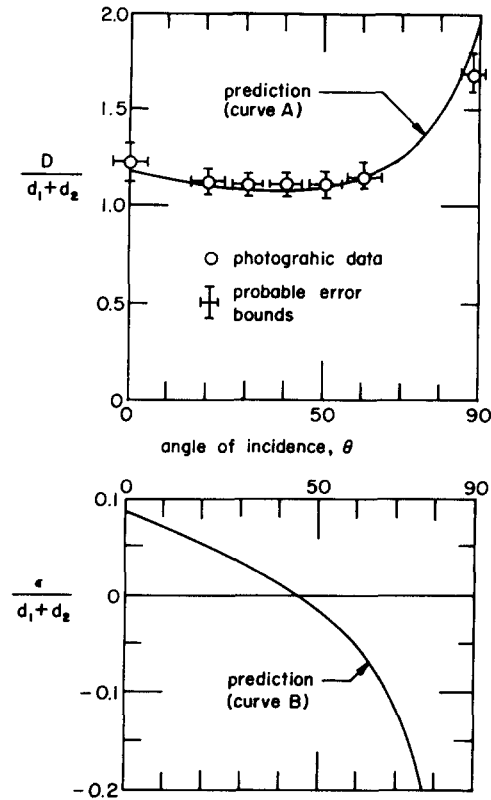


Figure 2. Diameter of water droplets (curve A), and location of their centers (curve B), as a function of the spacing between observed bright spots and the angle of illumination.

where n is the index of refraction ($n_{\text{H}_2\text{O}} = 1.33$). Eliminating β from [1] and [2] we obtain.

$$\tan \alpha = \frac{\cos \theta - n \sin(\pi/4 - \theta/2)}{n \cos(\pi/4 - \theta/2) - \sin \theta} \quad [3]$$

so we can compute α , given θ .

Now the true diameter of the droplet, D , can be obtained from a measured value of $(d_1 + d_2)$ since we know that:

$$d_1 = \frac{D}{2} \sin(\pi/4 + \theta/2) \quad [4a]$$

and

$$d_2 = \frac{D}{2} \cos(\alpha - \theta). \quad [4b]$$

The true position can also be obtained from an eccentricity, ϵ :

$$\epsilon \equiv (d_2 - d_1)/2 \quad [5]$$

although ϵ is normally quite small.

The dimensionless diameter of the droplet, $D/(d_1 + d_2)$ and the dimensionless eccentricity, $\epsilon/(d_1 + d_2)$ are plotted in figure 2. To obtain actual values of D and ϵ one must merely measure the distance between centers of the two spots that appear in a photograph and multiply this distance by either of the two preceding ratios.

To demonstrate the validity of the method we photographed a jet of water against a black

background with light directed against it at various angles. The distance $(d_1 + d_2)$ was measured between the centers of the resulting illuminated stripes down the jet. Enough background light could be tolerated in the steady jet configuration to provide faint illumination of the outside profile as well. Thus observations of $(d_1 + d_2)$ and D were provided for comparison with the calculations in figure 2.

These data verify the calculations of $D/(d_1 + d_2)$ within their probable error, which is indicated in the figure. The error arises both from measuring the photographs and from positioning the light. Since ϵ is on the order of magnitude of the error we have not tried to measure it.